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ORIGINAL ARTICLE

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Mechanothermodynamical system and its behavior

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Abstract An attempt to formulate the basic principles of mechanothermodynamics of systems is made. The notion of tribo-fatigue entropy generated in a mechanical system is introduced similarly to thermodynamic entropy governed by matter and energy exchange. Interrelation of motion, damage, and information is found. It is shown that motion generates new information in the system if its damageability index is non-zero. The information is positive if the system is hardened and negative if it softened. It leads one to perceiving the principal feature of interaction of irreversible damages (effective energies and entropy) generated by effects of different nature (mechanical loads, heat flows, etc.), viz. dialectic character of interaction. Two principles of thermodynamics are formulated.

Keywords Mechanothermodynamics · Tribo-fatigue · Entropy · Information · Interaction

1 Introduction

An attempt to formulate *the fundamentals of mechanothermodynamics of systems* has been made in the given work. The sequence of our analysis is as follows. First we introduce *the notion of tribo-fatigue entropy* generated in a mechanical system similar to *thermodynamic entropy* determined by energy and substance exchange. The radical difference of these notions is the following: *thermodynamic entropy* is a characteristic of energy dissipation, while *tribo-fatigue entropy* is a *characteristic of its absorption, hence, damage of moving and deformable solids*. Combining these concepts allows general outlines of *mechanothermodynamics* in terms of entropy to be constructed. To understand *the evolution of a system*, one need to establish a relationship between *motion, damage, and information*. It is shown that *the motion generates new information* in a system *if its damageability index is non-zero*; *information appears to be positive when the system is hardened and negative when it is softened*. This gives an impetus to perceive *the fundamental peculiarity of interaction of irreversible damages* (effective energy, entropy) of different nature (mechanical loads, thermal flows, etc.): the interaction reveals *the dialectic character* (so-called Λ-*interactions*). It has appeared that *the damageability is the fundamental physical property (and duty) of a mechanothermodynamic system* and Λ-interactions

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determine its evolution from the viewpoint of damageability having regard to diverse and complex processes of hardening–softening ($\Lambda \gtreqless 1$).

Striving for generalization is natural for a researcher because it is the way of perceiving the unknown. In conclusion, *an attempt has been made to consider the evolution of any systems*, including *the Universe, from a new viewpoint.* Two principles of *mechanothermodynamics have been formulated:*

1. damageability of everything that exists has no conceivable limits;
2. flows of effective energy (entropy) conditioned by sources of different nature interact dialectically ($\Lambda \gtreqless 1$).

2 Tribo-fatigue entropy

To describe the state of thermodynamic systems, the following functions of internal energy U and entropy S

$$U = U(T, V, N_k) \quad \text{or} \quad S = S(T, V, N_k) \tag{1}$$

are used where temperature T, volume V, and number of moles of chemical components N_k are the macroscopic state variables.

In the general case of an open system, the change dU of internal energy U is presented [1] as

$$\mathrm{d}U = \mathrm{d}Q + \mathrm{d}A + \mathrm{d}U_{\mathrm{sub}} = T\mathrm{d}S - p\mathrm{d}V + \sum_1^n \mu_k \mathrm{d}N_k, \tag{2}$$

where dQ is the amount of heat; dA is the amount of mechanical energy; dU_{sub} is the amount of matter the system exchanged with the environment for a time interval dt; p is the pressure in continuum corresponding to thermodynamic force; μ_k are the chemical potentials. Max Planck especially emphasized that in formula (2) dU is an infinitely small difference, where as dQ, dA, and dU_{sub} are infinitely small amounts.

From (2) it follows that the entropy increment

$$\mathrm{d}S = \frac{\mathrm{d}U + p\mathrm{d}V}{T} - \frac{1}{T}\sum_1^n \mu_k \mathrm{d}N_k. \tag{3}$$

can be presented as a sum of its change $\mathrm{d}_e S \gtreqless 0$ due to the exchange of system with energy and matter with the environment and the change $\mathrm{d}_i S \geq 0$ due to irreversible processes running inside the system:

$$\mathrm{d}S = \mathrm{d}_e S + \mathrm{d}_i S. \tag{4}$$

Thus, in thermodynamics the entropy S is a measure of irreversible dissipation of energy [2] that characterizes the system state from the viewpoint of its internal ordering or its structure.

Equations (2) and (3) do not take into account many processes, for example, the internal energy change at damage of moving and deformable solids and tribo-fatigue systems [3]. Matter exchange is considered only as a result of such processes as diffusion and chemical reactions, whereas matter exchange at surface wear and volume (e.g., wear-fatigue) damage is not allowed for. That is why the problem on evaluating the entropy arises in relation to numerous phenomena of damageability. Such phenomena are characteristic of the systems with moving and deformable objects.

According to the generalized concepts [3,4], damage is changes in composition, construction, structure, size, shape, volume, continuity, mass, and hence, in the corresponding physical-chemical, mechanical, and other properties of an object; finally, damage is related to breaking the continuity and integrity of a body up to its decomposition (e.g., into atoms) [4]. Thus, the damageability is treated as a fundamental property (and duty) of moving and deformable systems [3,4], and damage is considered as a specific type of fracture, i.e., corresponding fault of their continuity and integrity.

In tribo-fatigue, it is shown [3] that for tribo-fatigue systems the irreversible damageability ω_Σ is the function of the effective mechanical U_M^{eff}, thermal U_T^{eff}, and electrochemical U_{ch}^{eff} energies. Here we distinguish the mechanical energy due to the change in body sizes (U_σ^{eff}) and the mechanical energy due to the change in the body shape (U_τ^{eff}):

$$\begin{aligned}\omega_\Sigma &= \omega_\Sigma\left(U_\sigma^{\mathrm{eff}}, U_\tau^{\mathrm{eff}}, U_T^{\mathrm{eff}}, U_{ch}^{\mathrm{eff}}\right)\\ &= \omega_\Sigma(\sigma_{ij}, \varepsilon_{ij}, T_\Sigma, \nu_{ch}(m_\nu), \Lambda_{\sigma/p}, \Lambda_{T/M}) = \omega_\Sigma\left(U_\Sigma^{\mathrm{eff}}\right).\end{aligned} \tag{5}$$

Here the Λ-functions characterize the interaction of damages caused by different loads (force and contact-friction denoted by the subscript σ/p; thermal and mechanical by the subscript T/M), T_Σ is the temperature caused by all heat sources, $U_\Sigma^{\text{eff}} = U_\Sigma^{\text{eff}}(\sigma_{ij}, \varepsilon_{ij}, T_\Sigma, \nu_{ch}(m_\nu), \Lambda_{\sigma/p}, \Lambda_{T/M})$ is total effective energy.

In (5) the known relationships between energy and the corresponding force factors are taken (σ_{ij} and ε_{ij} are the stress and deformation tensors, ν_{ch} is the rate of electrochemical processes with regard to the material properties (m_ν)). The energy directly spent for formation and development of irreversible damages is called effective energy, i.e., U^{eff} is the absorbed part of the energy supplied to the system [3]. The methods of its determination are outlined in papers [3,4] which provide the formulas for estimation of ω_Σ under different operating conditions of tribo-fatigue systems. According to (5), the damageability ω_Σ is a thermomechanical function since it takes into account both the force factors and the temperature T_Σ caused by all heat sources.

As a rule, irreversible damages appear and are accumulated not in the entire (geometrical) volume of a deformable body, but only in some finite region of it with the critical state; this region is called the dangerous volume. The model of a body with a dangerous volume is developed in [5], and in [3,6] it is generalized for tribo-fatigue systems. A tribo-fatigue system is any mechanical system which takes up and transfers a repeatedly alternating load with the process of friction in any of its manifestations [3] occurring simultaneously.

As internal irreversible damages of thermomechanical nature originate due to the effective energy changes in the dangerous volume $W_{P\gamma}$ of the system, in the general case we have

$$\mathrm{d}U_\Sigma^{\text{eff}} = \gamma_1^{(w)}\, \omega_\Sigma\, \mathrm{d}W_{P\gamma}, \tag{6}$$

where $\gamma_1^{(w)}$ is the stress (pressure) that causes the damage of the unit dangerous volume ($W_{P\gamma} = 1$).

Then, according to (2)–(4), it is possible to introduce the notion of tribo-fatigue entropy, whose change is

$$(\mathrm{d}_i S)_{TF} = \frac{\gamma_1^{(w)}}{T_\Sigma}\, \omega_\Sigma\, \mathrm{d}W_{P\gamma}. \tag{7}$$

Thus, the tribo-fatigue entropy serves as a measure of irreversible absorption of the energy U_Σ^{eff} in the dangerous volume $W_{P\gamma}$ of the tribo-fatigue system.

Now consider an open thermodynamic system with a damageable solid; it is a mechanothermodynamic system. The entropy increment in such a system is obviously determined by the sum of thermodynamic entropy (3) and tribo-fatigue entropy (7):

$$(\mathrm{d}S)_T + (\mathrm{d}_i S)_{TF} = \frac{\mathrm{d}U + \Delta p \mathrm{d}V}{T} - \frac{1}{T}\sum_1^n \mu_k \mathrm{d}N_k + \frac{\gamma_1^{(w)}}{T_\Sigma}\, \omega_\Sigma\, \mathrm{d}W_{P\gamma}. \tag{8}$$

Here thermodynamic entropy (3) has a subscript T, in this case it is taken into account that $\Delta p \mathrm{d}V = (p_M - p)\mathrm{d}V$, $p_M \mathrm{d}V$ is the mechanical energy supplied to the system from the environment. If $\omega_\Sigma = 0$, then (8) reduces to (2). Tribo-fatigue entropy (7) is denoted by the subscript *TF*.

Equation (8) of the mechanothermodynamic state is radically different from Eq. (3) of the thermodynamic state: the first permits the analysis of any state of the system, including $A-$, $B-$, $C-$, $D-$, and E-states of damage (Table 1) [4] since in the general case $0 \le \omega_\Sigma \le \infty$ [3,4]. Hence, according to (8), exactly the growth of tribo-fatigue entropy production (7) due to the thermomechanical state of the system can cause both its damage and decomposition; thermodynamic Eq. (3) does not concern such states. The problem on the critical and supercritical entropy levels caused by the damage and failure of the systems remains to be investigated [1,7].

It should be noted that in continuum mechanics [8,9] the stress tensor is expanded into two components:

$$\sigma_{ij} = \sigma_{ij}^{(c)} + \sigma_{ij}^{(d)},$$

where the superscript (c) denotes the tensor of conservative stresses and the superscript (d) the tensor of dissipative stresses.

Then, the appropriate energy analysis yields the following thermomechanical function

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{1}{T}\frac{\mathrm{d}q}{\mathrm{d}t} + \frac{1}{\rho T}\, \sigma_{ij}^{(d)}\, \dot{\varepsilon}_{ij}, \tag{9}$$

Table 1 States of objects and their characteristics

A-state	Undamaged	$\omega_\Sigma = 0 = \omega_0$	↓ *A*-evolution: characteristic states of a system
B-state	Damaged	$0 < \omega_\Sigma < 1$	
C-state	Critical	$\omega_\Sigma = 1 = \omega_c$	
D-state	Supercritical	$1 < \omega^*_\Sigma < \infty$	
E-state	Decomposition	$\omega_\Sigma = \infty = \omega_\infty$	

where $\mathrm{d}q/\mathrm{d}t$ is the rate of the heat flux to the medium per unit mass; $\frac{1}{\rho}\sigma^{(d)}_{ij}\dot{\varepsilon}_{ij}$ is the energy dissipation rate per unit mass (ρ is the medium density).

Equation (9) is valid only for a continuous medium. If the continuity of a deformable solid is disturbed, it cannot be used, i.e., it is not capable of describing critical and supercritical states of the system, for example, according to Table 1. This is just the cardinal difference between Eqs. (9) and (8). If one takes into account that internal irreversible damage is a fundamental property and a duty of a moving and deformable system, then one may arrive at the notion of tribo-fatigue entropy (7), (9) as a measure of absorption of energy spent for generating and developing such damages in the mechanothermodynamic system. Generally, the latter is termed as an open (thermodynamic) system containing a moving and deformable solid. A specific feature of the mechanothermodynamic system, unlike the thermodynamic system, is that both thermodynamic and tribo-fatigue entropies are generated in it.

3 Interrelation of motion, damage, and information

As shown above, the processes of irreversible damage in a mechanothermodynamic system generate tribo-fatigue entropy. This means that, on the one hand, a relationship should exist between the motion and damage. On the other hand, it is obvious that the damage of a moving system alters its information state. Hence, there arises a general problem on searching the interrelation

$$\mathrm{Mot} \Leftrightarrow \mathrm{Inf} \Leftrightarrow \mathrm{Dam}, \tag{10}$$

where Mot denotes the motion, Inf the information, and Dam the damage.

Let us specify problem (10) consecutively.

First let us find a function describing the interrelation of motion and information, i.e.,

$$\mathrm{Mot} \Leftrightarrow \mathrm{Inf}. \tag{11}$$

In the general case the transfer function of motion is

$$\dot{X} = F(t, X, C), \tag{12}$$

where $X = (x_1, x_2, x_3, \ldots x_n)$ is the vector of system states in n-dimensional space, $C(t, X)$ is the vector function of control.

Let us find the solution of problem (11), as applied to non-controlled ($C = 0$) linear dynamic system. According to the theory of control, the velocity of the change of system state is the vector function of its state, and for such a system, mathematical model (12) becomes the simplest:

$$\dot{X} = BX, \tag{12a}$$

where the matrix of dimension $n \times n$ is

$$B = \begin{bmatrix} b_{11} & b_{12} & \ldots & b_{1n} \\ b_{21} & b_{22} & \ldots & b_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ b_{n1} & b_{n2} & \ldots & b_{nn} \end{bmatrix}$$

and b_{ij} are the coefficients influence of components x_j on components x_i.

Spur of matrix B is

$$\mathrm{tr}\, B = b_{11} + b_{22} + \cdots + b_{nn}.$$

For system (12a) O.T. Vavilov's functional of information transformation reduces to the following expression [10]:

$$\log_2 P(0, X(0)) - \log_2 P(T, X(T)) = k \int_0^T \operatorname{tr} B \, \mathrm{d}t, \tag{13}$$

where $k = \ln 2$ is the constant, $P(T, X(T))$ is probability density function defining position (state) X of a system at time T.

If in (13) it is assumed that $T = t$, then the consideration can be supplemented with the information function of the dynamic system

$$\Delta I = I_2 - I_1 = -k \cdot \operatorname{tr} B \cdot t. \tag{14}$$

Here the dimension of B should be such that ΔI would be determined in information units (bytes).

Exactly, function (14) is the solution of problem (11): it characterizes information transformation in the simplest dynamic system (12a) in going from one state to another.

Now for (10) to be defined concretely, let us study the damageability of the system in time. We consider that the main types of kinetic processes of damage $\omega_{\Sigma t}$ of an object (solid, tribo-fatigue system) can be described by the simplest power equation [3,5]

$$\omega_{\Sigma t} = \left[1 - \left(1 - \frac{t}{T_\otimes}\right)^h\right]^q, \tag{15}$$

where $T_\otimes$ is the life (durability); $h \geq 1$, $q \geq 1$ are the controlling parameters. If $h \geq 1$, $q = 1$, then the phenomena of material softening (the convex curve in Fig. 1) are dominating, while, on the contrary, at $q > 1$, $h = 1$ these are the phenomena of material hardening (the concave curve in Fig. 1). At $h = 1$, $q = 1$ the system is stab (the dotted line in Fig. 1). In the general case $h > 1$, $q > 1$ the processes of hardening–softening of the system are determined by the parameter ratio h/q and are described by more complex (S-shaped) curves.

For any fixed time moment $t/T_\otimes = \text{const}$, let us introduce a unique characteristic of systems – the damage index

$$\omega_j = \omega_{st} - \omega_{\Sigma t}, \tag{16}$$

where $\omega_{\Sigma t} = \omega_h$ or $\omega_{\Sigma t} = \omega_q$ is the damage level of a real system, and ω_{st} is the damage level of some ideal system corresponding to that of the real system.

Then, it turns out that the values of damage index (16) can belong to three characteristic classes: $\omega_j > 0$; $\omega_j < 0$ and $\omega_j = 0$ (Fig. 3). There are the same three classes are for information increment ΔI

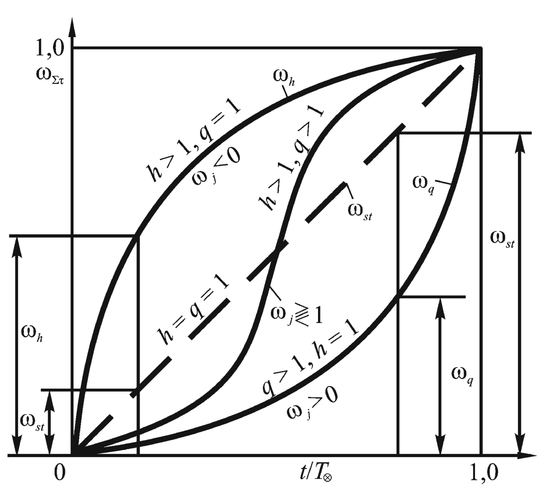

Fig. 1 Schemes of possible kinetic processes of irreversible damages

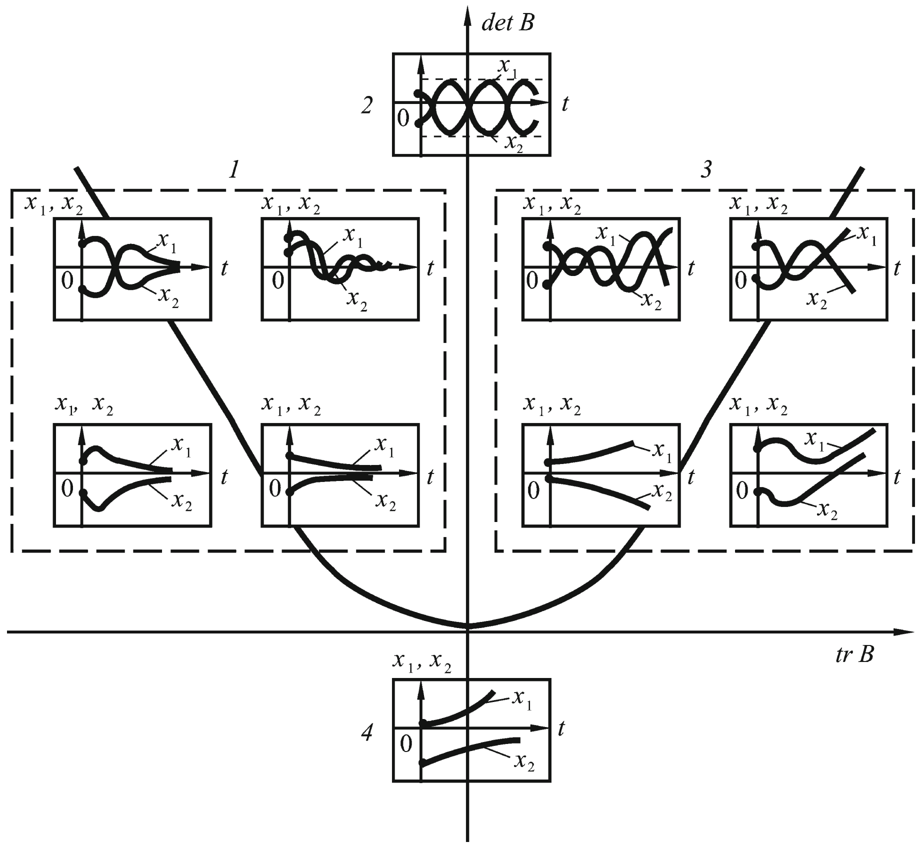

Fig. 2 Time diagrams of the second-order dynamic system: *1*— oscillating and asymptotic converging processes, *2*—continuous oscillating processes, *3*—oscillating and asymptotic diverging processes, *4*—unstab. processes

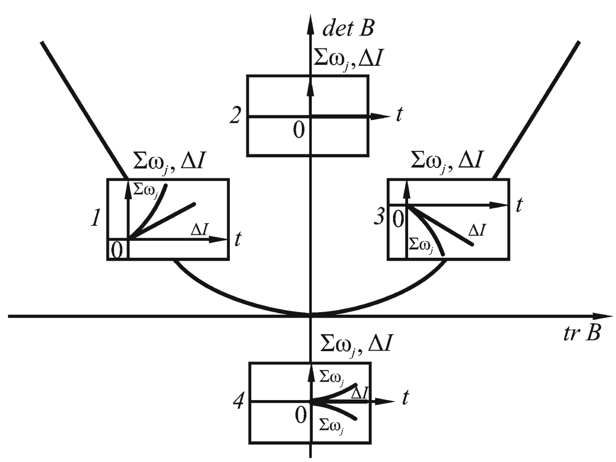

Fig. 3 Information increment and the changes of the damage index in the moving system

(Fig. 3). This means the existence of dependence between ω_j and ΔI. This dependence may be formulated as a first approximation in the following way

$$\Delta I(t) = -k \cdot \mathrm{tr}B \cdot t = a_S \Sigma\, \omega_j(t), \tag{17}$$

where a_S is the transition function between ΔI and $\Sigma\, \omega_j$. The form of function a_S has not yet been found, but its sense is that it transforms damages (15) into information function (14) through damage index (16).

This is just the simplest solution of problem (10).

Figures 2 and 3 present the graphical illustration of Eq. (17).

Table 2 Interrelation of motion, information, and damageability

Group (Figs. 1, 2)	Signs		Processes		Systems	Damageability		Information functions
	det B	tr B	Time	Physical		Processes	Index	
1	+	–	Converging (oscillating and asymptotic)	Irreversible	Dissipative, div $F(\bullet) < 0$	Hardening	$\omega_j > 0$	Linear positive
2	+	0	Oscillating continuous	Reversible	Conservative, div $F(\bullet) = 0$	Stab.	$\omega_j = 0$	Zero
3	+	+	Diverging (oscillating and asymptotic)	Irreversible	Nonconservative (dissipative), div $F(\bullet) > 0$	Softening	$\omega_j < 0$	Linear negative
4	–	0	Unstable	Irreversible	Conservative, div $F(\bullet) = 0$	Stab. with fluctuations of hardening–softening	$\omega_j \gtreqless 0$	Zero

Figure 2 presents schematically (in the frames) typical motions of system (12a). They are plotted in the x_1, x_2 coordinates in time; the arrows denote the motion directions. The entire set of the time diagrams (Fig. 2, ten diagrams) is located in the common plane, whose abscissa axis is the trend of the matrix B, and the ordinate axis is its determinant det B. One can plot the graph of the function $\det B = (\operatorname{tr} B)^2$; it is the parabola in Fig. 2. Then, the time diagrams of the second-order dynamic system are located relative to this parabola, as shown in Fig. 2. It is seen that four groups (1, 2, 3, and 4) of the diagrams are available, each of which lies in the characteristic zone of the plane depending on the ratio and signs of det B and tr B.

Figure 3 compares the time diagrams of system (12a) presented in Fig. 2 with the plots of the information function $\Delta I(t)$ and the function of accumulation of the damageability $\Sigma\, \omega_j(t)$. The notations of the data groups in Fig. 3 are the same as in Fig. 2.

The analysis of the data plotted in Figs. 2 and 3 yields three general conclusions.

First, the left branch of the parabola plotted in the det B—tr B coordinates illustrates the stab. Dynamic processes that generate a positive linear information function due to the developing of non-linear hardening of an object.

Second, the right branch of the parabola plotted in the det B —tr B coordinates corresponds to the unstable dynamic processes that generate a negative linear information function due to the developing of non-linear softening of an object.

Third, the vertex of the parabola plotted in the det B—tr B coordinates can be consistent with the radically different states of the system: (a) continuous oscillating processes (above the parabola vertex—on the det B axis)—new information is not generated here since the damageability index $\omega_j = 0$; (b) unstab. processes (below the parabola vertex—on the same det B axis), when a zero information function is again generated, and the damageability index $\omega_j \gtreqless 0$. To understand this contradiction, one needs to search some specific features of this instability that are fundamentally different from the instability of vibrational and asymptotic diverging processes.

Our available basic data on second-order dynamic systems are systematized in Table 2 taking into account that for dissipative systems the divergence of function (12) div $F(t, X, C) \neq 0$ and for conservative systems div $F(t, X, C) = 0$.

The analysis of Table 2 allows us to answer the above question: why does not any information arise as a result of an unstab. process? It appears because it is the case of a conservative system, for which the divergence div $F(t, X, C) = 0$ and $\omega_j = 0$. Hence, the motion does not initiate information if the divergence and so the damageability of the system are zero. In other words, a conservative system cannot produce new information.

Thus, the interrelation of motion, information, and damageability (Figs. 2, 3; Table 2) has been established by the simplest example. Its analysis permits one to make the following basic conclusions: the motion generates new information in the system if its damageability index is equal to zero; the information is positive, when the system is hardened, and is negative, when it is softened.

4 Principles of mechanothermodynamics

Earlier it has been asserted that the mechanothermodynamic state of a system can be in principle characterized by total changes in the dissipated and absorbed parts of energy, or entropy.

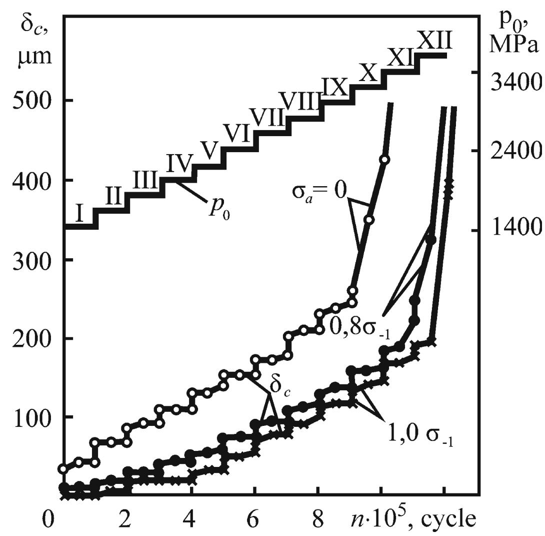

Fig. 4 Test results

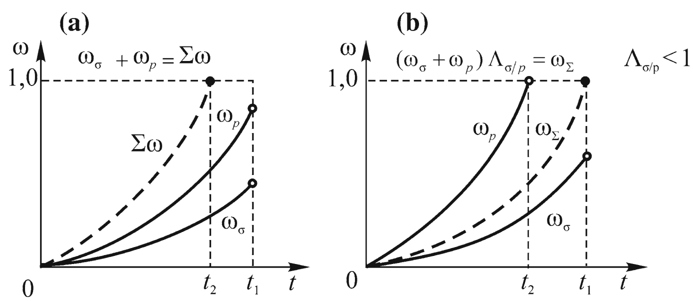

Fig. 5 Scheme illustrating the summation (**a**) and interaction (**b**) of damages

Let us mention here one important problem. In [3,11] it is shown that the damages due to different-nature loads (e.g., thermal and mechanical) do not exhibit the additivity properties; on the contrary, they have the property of non-linear interaction. The results on such interaction are described by the functions $\Lambda_{\sigma/p}$, $\Lambda_{T/M}$ (expression (5)). In [6,11] the Λ-functions were defined concretely for some deformable systems.

Let us support the aforesaid experimentally.

Comparative experiments were performed on the damage of a deformable system at rolling friction and complex loading: rolling friction + mechanical fatigue. In both cases, during tests the contact pressure p_0 was increased stepwise over its range (Fig. 4, stages I, II, ..., XII). When testing the shaft/roller system the approach δ_c of the axes of this pair of the elements was measured under the rolling friction conditions (when the amplitude of cyclic stresses $\sigma_a = 0$) and under contact-mechanical fatigue conditions (at $\sigma_a = 0.8\,\sigma_{-1}$ and $\sigma_a = 1.0\,\sigma_{-1}$ where σ_{-1} is the fatigue limit). It can been seen (Fig. 4) that the accumulation of complex wear-fatigue damages essentially slows down in comparison with the damage at rolling friction; in this case, the range of contact pressure corresponding to normal friction widens approximately by 14%. Based on these experimental data, let us discuss the qualitative difference between the processes of summation and interaction of damages considering that limiting state is reached when measure of damage ω becomes critical, i.e., ω=1.

Let damages due to contact (ω_p) and non-contact (ω_σ) loads will be accumulated for the time t_1 in the manner shown in Fig. 5, *a*: the critical state is achieved by none of these criteria ($\omega_p \ll 1.0$; $\omega_\sigma \ll 1.0$). If the damages are summed up $(\omega_p + \omega_\sigma) = \Sigma\,\omega$, then in the case of wear-fatigue tests the critical state ($\Sigma\,\omega = 1.0$) will be reached for the time $t_2 < t_1$. However, such a prediction appears to be apparently incorrect, as applied to the experimental data in Fig. 4. If it is taken into account that the damages caused by contact and non-contact loads interact

$$(\omega_p + \omega_\sigma)\Lambda_{\sigma/p} = \omega_\Sigma, \tag{18}$$

so that under the analyzed conditions interaction function $\Lambda_{\sigma/p} < 1$, then the scheme adequately illustrating the experimental data in Fig. 4 looks like the one shown in Fig. 5b. Fig. 5b shows that at rolling friction the critical state is reached by ω_p for the time t_2, whereas at mechanical fatigue (ω_σ) it does not occur even for $t_1 \gg t_2$. Under the conditions of the wear-fatigue tests characterized by measure of damage ω_Σ the life (t_1) turns out to be larger than the one at rolling friction (t_2).

Since the irreversible damageability is the function of the effective energy absorbed in the system (expression (5)), from (18) the general conclusion can be made: at wear-fatigue damage, effective energies due to contact (U_p^{eff}) and non-contact (U_σ^{eff}) loads are not summed up, they interact dialectically. Now one can write the principle of interaction of the effective energy components in the tribo-fatigue system [3,6,13]

$$(U_\sigma^{\text{eff}} + U_p^{\text{eff}})\Lambda\left(\omega_\sigma \rightleftarrows \omega_p\right) = U_\Sigma^{\text{eff}}, \quad \Lambda \gtreqless 1. \tag{19}$$

In case considered by (19) effective energies are proportional [3] to the squares of the acting cyclic stresses σ^2 and contact pressure p^2:

$$U_\sigma^{\text{eff}} = A_\sigma \sigma^2,$$
$$U_p^{\text{eff}} = A_p p^2.$$

The technique of calculation of coefficients A_σ and A_σ and interaction function Λ is described in [3].

According to (19), the result on the $\left(U_\Sigma^{\text{eff}}\right)$ interaction of damages ($\omega_\sigma \rightleftarrows \omega_p$) and, hence, of energies is governed both by the loading conditions and the direction of the processes of hardening–softening ($\Lambda \gtreqless 1$) [3]. From (19) it follows that at $\Lambda(\omega_\sigma \rightleftarrows \omega_p) = 1$ the particular case of interaction of the effective energies (and, hence, of the damages) —their summation—is possible.

Developing the views on interaction of effective different-nature energies according to model (19) [3,6, 11,12] yields a diversity of new conclusions since it involves a physically clear result: real damage and failure of systems. Four first surprises of tribo-fatigue ([11]) could be neither understood, nor described without the knowledge of principle (19).

According to the available data [1,7], when the behavior of thermodynamic systems was analyzed the problem of the interaction of the dissipated part of different-nature energies was not stated. It is natural that possible interaction of entropy produced by mechanothermodynamic forces and flow has not been investigated when different irreversible processes ((3), (8), (9)) are realized. But since the entropy and energy relation $S(U)$ is fundamental ((1) and (2)), resting upon the above-said in the general case of the analysis of the mechanothermodynamic state of systems sum (8) of thermodynamic and tribo-fatigue entropies should be written having regard to possible Λ-interactions:

$$S_{\text{total}}\,(t) = (S_T\,(t) + S_{TF}\,(t))\,\Lambda_{T/TF}. \tag{20}$$

As mentioned, the fundamentals of the theory of Λ-interactions of irreversible damages in tribo-fatigue systems have been formulated and to some extent developed up to now [3,6,12]. Creating the theory of irreversible Λ-interactions in mechanothermodynamic systems is waiting for its researcher. But Eq. (20) in combination with the results reported in [3,6,12] enables one to analyze in principle the mechanothermodynamic state of systems.

To analyze let us use four parameters (Table 3).

Now it is possible to plot, for example, Fig. 6. For the definite parameter ratios presented in Table 3 Eq. (20) predicts various and complex trajectories of entropy. In the course of evolution the system can enter, for example, in the stab. and equilibrium states and come out from them as many times as possible under the particular conditions of its existence; the observed points A_1, A_2 of the system can approach each other and come apart

Table 3 Parameters for the mechanothermodynamic state of different systems

Parameter	Characteristic
div $F(\bullet) \gtreqless 0$	Relative motion of physical points of matter or elements of a system (converging, diverging, and other processes)
$\omega_j \gtreqless 0$	Nature of irreversible damageability (hardening, softening, etc.)
$\Lambda \gtreqless 1$	Direction and intensity of interaction of irreversible damages of any nature
$\pm\Delta I$	Information changes in the process of motion and damage of a system

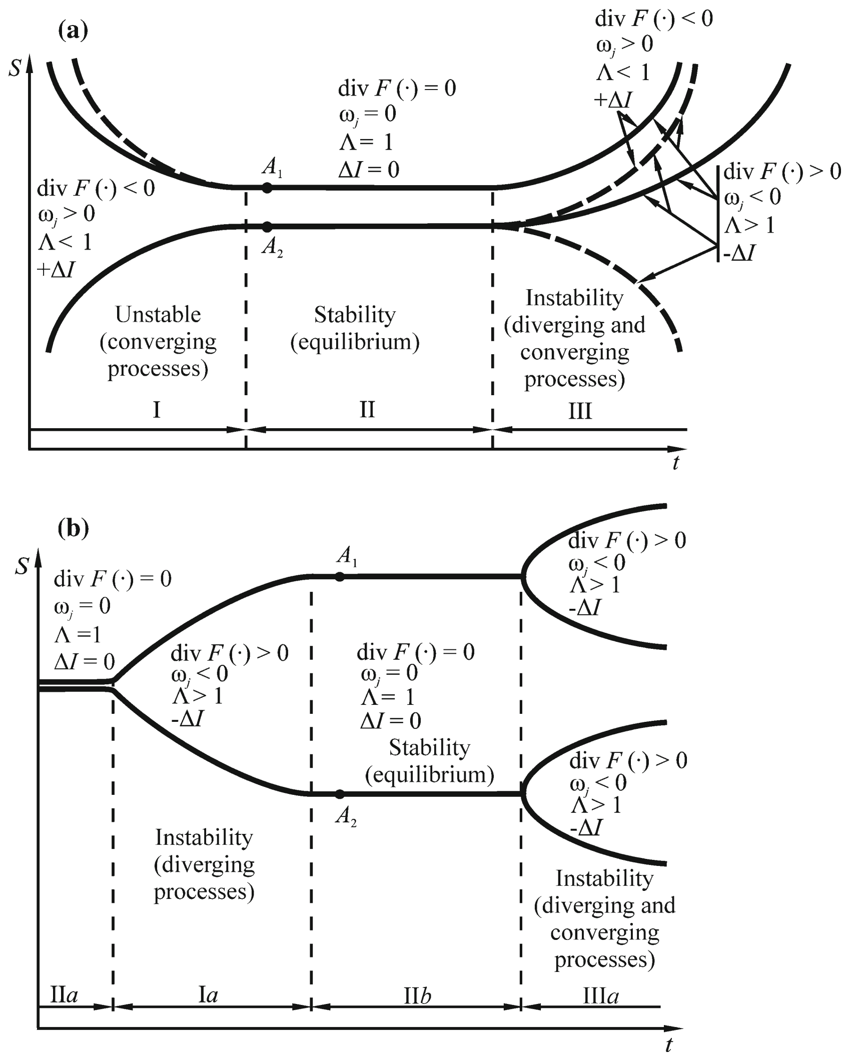

Fig. 6 Possible transitions of the system from the unstab. to the stab. state and vice versa (**a**), and the onset of bifurcations (**b**)

or move practically parallel; the system can undergo bifurcations and other (more complex) transformations. Figure 6b illustrates that bifurcations are peculiar to softening systems with the negative information function. Then, it is natural that the question arises: what is the difference between the mechanothermodynamic and thermodynamic processes?

An answer to this question is illustrated by Fig. 7. Here the solid lines stand for the predicted behavior of the thermodynamic system, for which in (20) it is assumed that $S_{TF} = 0$ and $\Lambda_{T/TF} = 1$; let the entropy S_T of such a system tend to some (for example, local) maximum. The behavior of the mechanothermodynamic system is shown in Fig. 7 by the dotted lines, assuming that in (20) we have $S_T \neq 0$ and $\Lambda_{T/TF} > 1$. The initial state of both systems is assumed to be identical (the points A_1, A_2). The fate of the system in the both cases is determined by the intensity of numerous irreversible internal processes caused by a diversity of reasons. But it will be fundamentally different for the comparable systems.

On the other hand, the trajectory of the mechanothermodynamic state ($S_T + S_{TF}$) cannot coincide with that of the thermodynamic state (S_T) since in the first case there appears a non-zero addition of tribo-fatigue entropy ($S_{TF} > 0$). This motivates quantitative differences in the trajectories of the systems to be compared. On the other hand, the principal difference is seen in their behavior: when the entropy of the thermodynamic system attains, for example, a local maximum (equilibrium state), the mechanothermodynamic system can have no such maximum—it will be in the non-equilibrium state. This is observed in the cases of converging (Fig. 7a) and diverging (Fig. 7b) processes of motion (Fig. 2) and for systems hardening and softening in time,

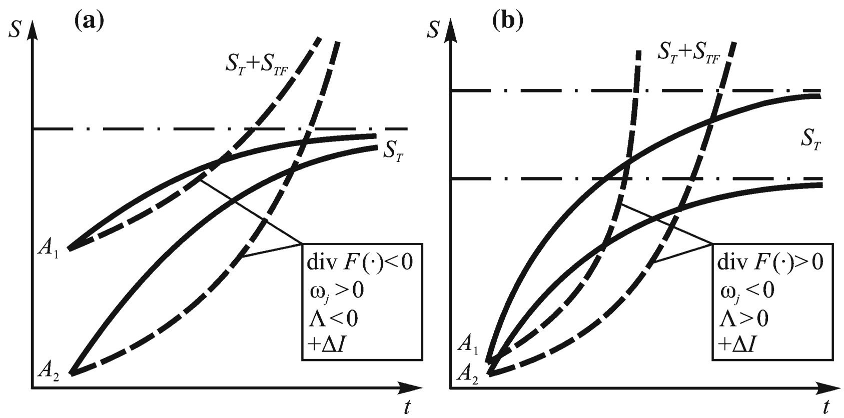

Fig. 7 Evolution of the thermodynamic (S_T) or mechanothermodynamic ($S_T + S_{TF}$) state of the system (A_1, A_2): **a** oscillating and asymptotic converging processes; **b** oscillating and asymptotic diverging processes

in which new positive or negative information is generated (Fig. 3). Work [6] contains some generalizations regarding a comparative behavior of thermodynamic and mechanothermodynamic systems.

5 Entropy calculation

Consider the example of entropy calculation for the tribo-fatigue system consisting of friction pair with the elliptic contact of the ratio between smaller b and bigger a semi-axes $b/a = 0.574$. One of the elements of the friction pair is loaded by non-contact bending. An example of such an element is the shaft in the roller/shaft tribo-fatigue system.

Specific damage ω in (7) of elementary dW which can be presented as a ratio between current parameter φ_{ij} of mechanical state (stresses and strains) of a system and its limiting value $\varphi_{ij}^{(\text{lim})}$.

Such ratios may be of two kinds: dimensional and dimensionless for stress tensor

$$\omega = \sum_{i,j=1}^{3} \left(\sigma_{ij} - \sigma_{ij}^{(\text{lim})}\right) \quad \text{and} \tag{21a}$$

$$\omega = \sum_{i,j=1}^{3} \left(\sigma_{ij} \,/\, \sigma_{ij}^{(\text{lim})}\right), \tag{21b}$$

for stress intensity

$$\omega = \sigma_{\text{int}} - \sigma_{\text{int}}^{(\text{lim})} \quad \text{and} \tag{22a}$$

$$\omega = \sigma_{\text{int}} \,/\, \sigma_{\text{int}}^{(\text{lim})}, \tag{22b}$$

where

$$\sigma_{\text{int}} = \frac{\sqrt{2}}{2}\sqrt{(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2 + 6\left(\sigma_{12}^2+\sigma_{23}^2+\sigma_{13}^2\right)}$$

and energy

$$\omega = U - U^{(\text{lim})} \quad \text{and} \tag{23a}$$

$$\omega = U/U^{(\text{lim})} \tag{23b}$$

where strain energy is

$$U = \sum_{i,j=1}^{3} \int_{\varepsilon_{ij}=0}^{\varepsilon_{ij}} \sigma_{ij}\, \mathrm{d}\,\varepsilon_{ij}, \tag{24}$$

that becomes

$$U = \sum_{i,j=1}^{3} \sigma_{ij}\, \varepsilon_{ij}, \tag{25}$$

in case of the considered linear dependence between stresses σ_{ij} and strains ε_{ij}.

According, for example, to [3], dangerous volume in a solid is the three-dimensional set of points (elementary volumes dV) where acting stresses (strains), stress intensity, or strain energy surpass the limiting and therefore produce damage:

$$\begin{aligned} W_{ij} &= \left\{ \mathrm{d}V / \varphi_{ij} \geq \varphi_{ij}^{(\mathrm{lim})}, \mathrm{d}V \subset V_k \right\}, \quad i, j = x, y, z. \\ W_{\mathrm{int}} &= \left\{ \mathrm{d}V / \varphi_{\mathrm{int}} \geq \varphi_{\mathrm{int}}^{(\mathrm{lim})}, \mathrm{d}V \subset V_k \right\}, \\ W_U &= \left\{ \mathrm{d}V / U \geq U^{(\mathrm{lim})}, \mathrm{d}V \subset V_k \right\} \end{aligned} \tag{26}$$

where V_k. is working volume.

Since $\omega\,\mathrm{d}W$ should have energy dimension formulas (21–23a) may be used directly. In case formulas (21–23b) are used it is necessary apply dimension factor $\gamma_1^{(w)}$.

Consider the following expression for calculation of energy dangerous volume W_U and tribo-fatigue entropy S_U

$$\begin{aligned} W_U &= \iiint\limits_{W_U\,(U \geq U^{(\mathrm{lim})})} \mathrm{d}W_U \\ S_U &= \iiint\limits_{W_U\,(U \geq U^{(\mathrm{lim})})} \mathrm{d}S_U \mathrm{d}W_U \end{aligned} \tag{27}$$

where according to (23b) $dS_U = \omega / T = \left(U - U^{(\mathrm{lim})}\right) / T$.

Note that integrals (27) are calculated only in the dangerous volume where $U \geq U^{(\mathrm{lim})}$ and therefore energy is absorbed to produce the damage unlike the volumes where energy is just dissipated if $U < U^{(\mathrm{lim})}$. Calculation of these integrals due to the complexity of the surface bounding the dangerous volume was performed numerically using Monte-Carlo method.

The following input data were used in calculations of entropy for elliptic contact if contact pressure is

$$p^{(n)}(x, y) = p_0^{(c)} \sqrt{\left(1 - x^2/a^2 - x^2/b^2\right)}$$

and the ratio between the bigger a and the smaller b semi-axes of contact ellipse is $b/a = 0,574$:

$$\begin{aligned} p_0^{(c)} &= \sigma_{zz}^{(n)}(F_c)\Big|_{x=0,y=0,z=0} = 2,960\,\mathrm{MPa}, \\ p_0^{(c,\mathrm{lim})} &= p_0\left(F_c^{(\mathrm{lim})}\right) = 888\,\mathrm{MPa} = 0.3 p_0^{(c)} \end{aligned} \tag{28}$$

where $p_0^{(c)}$ is the maximum contact stress under the action of force F_c, $p_0^{(c,\mathrm{lim})}$ is the contact fatigue limit (maximum contact stress under the action of limiting force $F_c^{(\mathrm{lim})}$ obtained in the course of mechano-rolling fatigue tests described in [3].

The criterion of the limiting state in these tests was the limiting approach of the axes in the tribo-fatigue system (100 μm). The test base equaled to $3 \cdot 10^7$ cycles.

Calculations of the three-dimensional stress-strain state in the neighborhood of elliptic contact for $b/a =$ 0.574 [14,15] show that maximum value of strain energy U is related to the maximum contact pressure $p_0^{(c)}$ in the following way

$$U = \max_{dV}\left[U\left(F_c, dV\right)\right] = 0,47 p_0^{(c)}. \tag{29}$$

Therefore the limiting value of strain energy $U^{(\lim)}$ for the action of limiting force $F_c^{(\lim)}$ is

$$U^{(\lim)} = \max_{dV}\left[U\left(F_c^{(\lim)}, dV\right)\right] = 0,47 p_0^{(c,\lim)}. \tag{30}$$

Maximum stresses σ_a caused by non-contact bending in the contact area in calculations were the following $-0.34 \le \sigma_a / p_0^{(c)} \le 0.34$.

Tangential tractions are directed along the greater semi-axis of contact ellipse:

$$p^{(\tau)}(x, y) = -f p^{(n)}(x, y) = -f p_0^{(c)} \sqrt{\left(1 - x^2/a^2 - x^2/b^2\right)}$$

Distribution of entropy increment calculated according to (23a) shown in Figs. 8, 9, 10, and 11 could be considered as the characteristic of probability of appearance of local damage (initial cracks). The higher is the entropy increment in a point of the dangerous volume the higher is the probability of damage (crack) initiation in this point. Magnitudes of the dangerous volumes and entropy are the integral damage indicators (possible number of cracks and their sizes) of a body or a system.

According to Figs. 8, 9, 10, and 11 for $p_0 = p_0^{(c)}$ and friction coefficient $f = 0.2$ maximum entropy increment is in the middle of contact surface.

Maximum of entropy increment under the joint action of contact pressure and tangential tractions (friction) $dS_U^{(n+\tau)}$ increases by approximately 30% comparing to the maximum of entropy increment $dS_U^{(n)}$ under the action of contact pressure only. Joint action of friction and tensile bending stresses increase $dS_U^{(n+\tau+b)}$ by approximately 30% and compressive bending stresses increase $dS_U^{(n+\tau-b)}$ by approximately 60% comparing to the maximum of entropy increment $dS_U^{(n)}$.

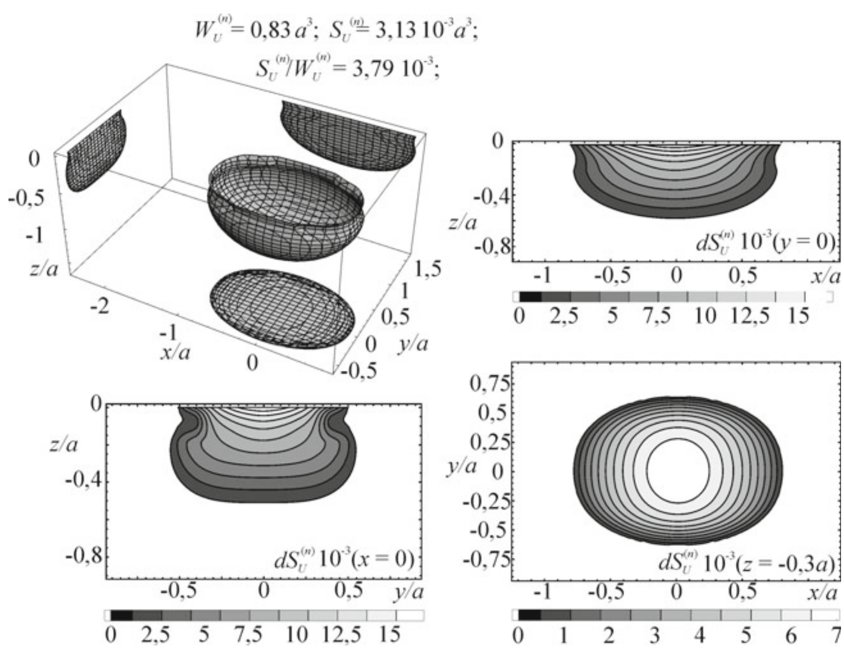

Fig. 8 Energy dangerous volume and its sections with distributions of entropy increment for pure elliptic contact

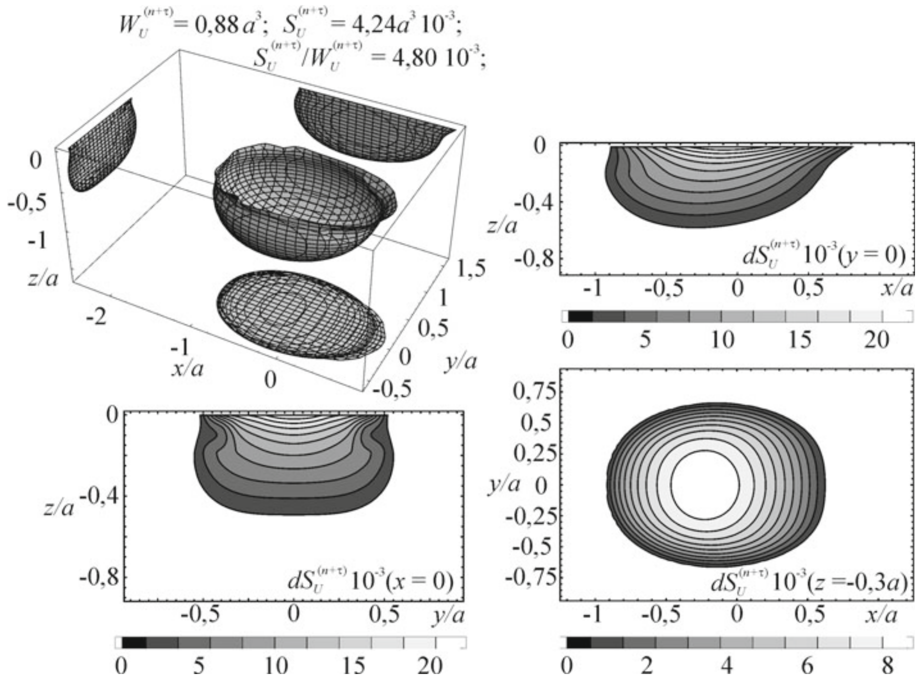

Fig. 9 Energy dangerous volume and its sections with distributions of entropy increment for friction pair

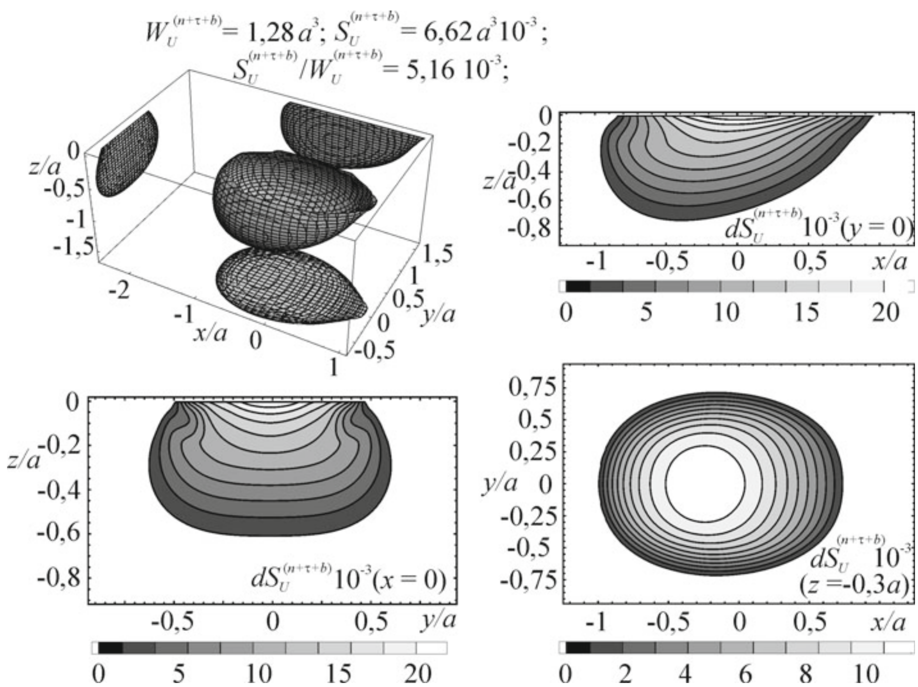

Fig. 10 Energy dangerous volume and its sections with distributions of entropy increment for tribo-fatigue system with tensile stresses $\sigma_a / p_0^{(c)} = 0.34$ in contact area caused by non-contact bending

If friction is applied to the system, then the magnitudes of dangerous volume $W_U^{(n+\tau)}$, entropy $S_U^{(n+\tau)}$, and average entropy $S_U^{(n+\tau)} / W_U^{(n+\tau)}$ are increased by approximately 6, 35, and 27 % comparing to $W_U^{(n)}$, $S_U^{(n)}$ and $S_U^{(n)} / W_U^{(n)}$, respectively.

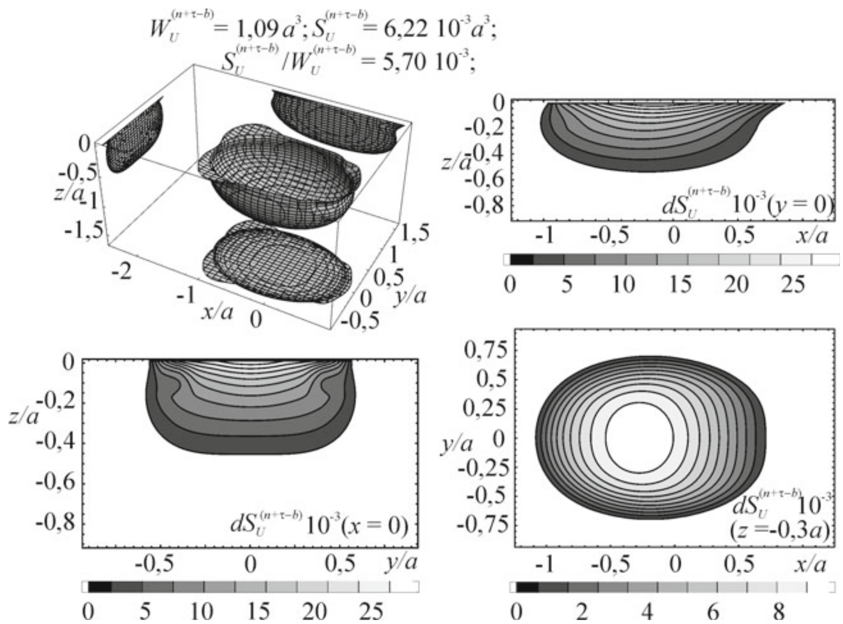

Fig. 11 Energy dangerous volume and its sections with distributions of entropy increment for tribo-fatigue system with compressive stresses $\sigma_a / p_0^{(c)} = -0.34$ in contact area caused by non-contact bending

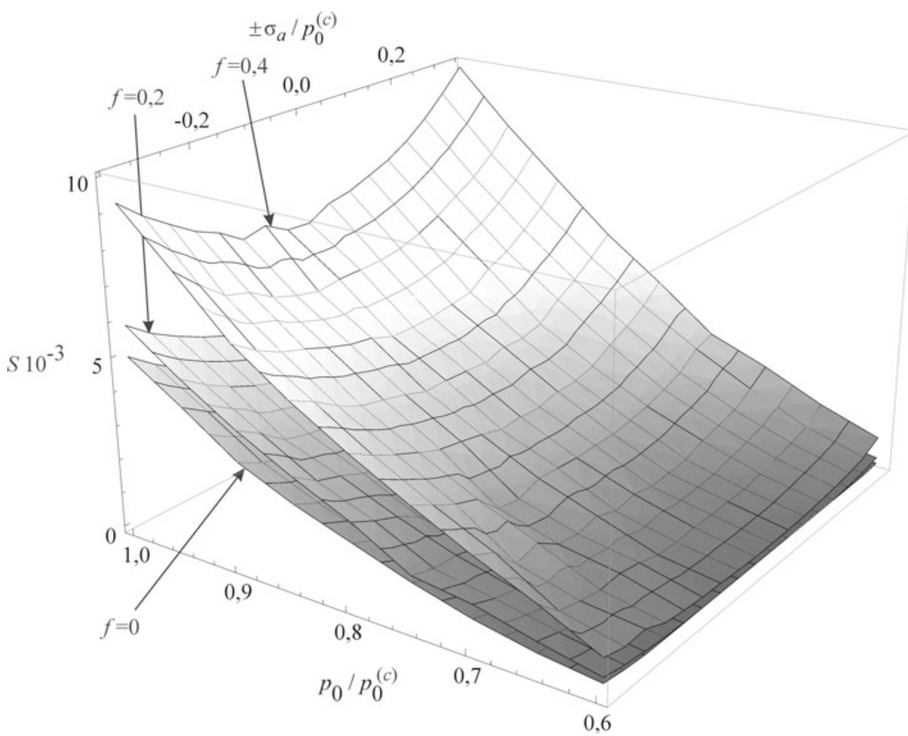

Fig. 12 Dependence of entropy on contact and non-contact stresses

If both friction and tensile bending stresses are applied to the system, then the magnitudes of dangerous volume $W_U^{(n+\tau+b)}$, entropy $S_U^{(n+\tau+b)}$, and average entropy $S_U^{(n+\tau+b)} / W_U^{(n+\tau+b)}$ are increased by approximately 54, 112, and 36 % comparing to $W_U^{(n)}$, $S_U^{(n)}$ and $S_U^{(n)} / W_U^{(n)}$, respectively.

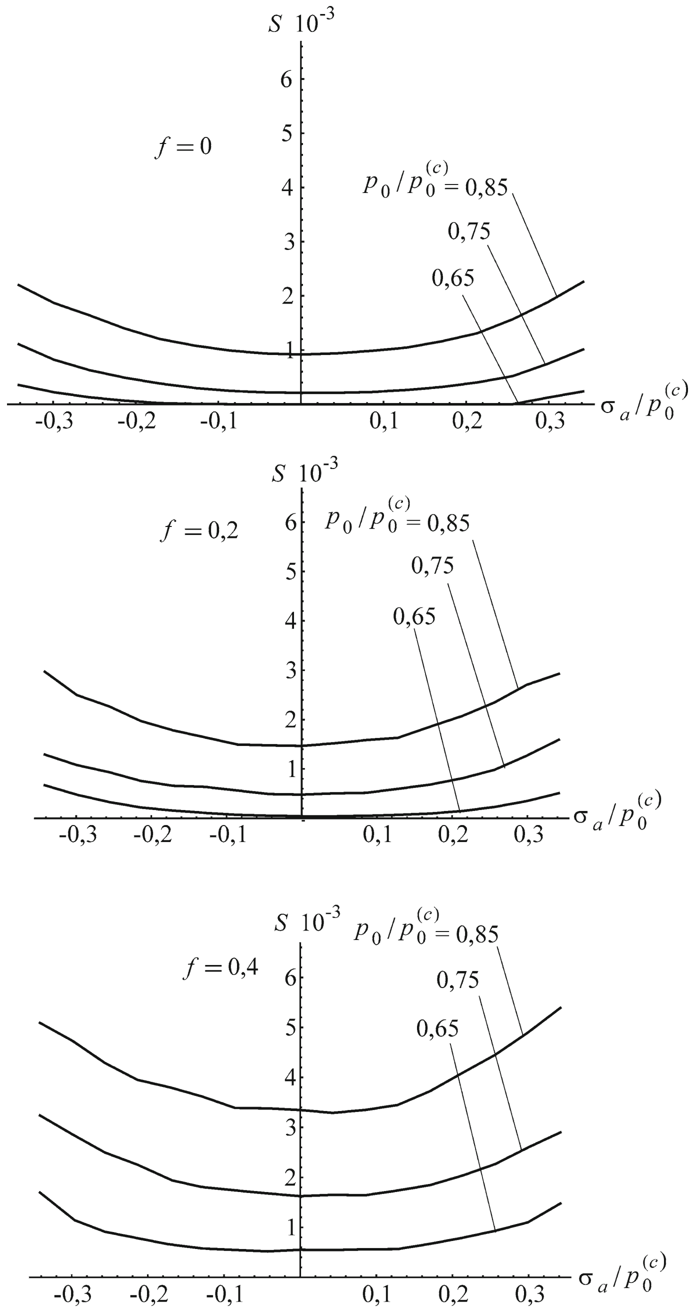

Fig. 13 Dependence of entropy and non-contact stresses

If both friction and compressive bending stresses are applied to the system, then the magnitudes of dangerous volume $W_U^{(n+\tau-b)}$, entropy $S_U^{(n+\tau-b)}$, and average entropy $S_U^{(n+\tau-b)}/W_U^{(n+\tau-b)}$ are increased by approximately 31, 98, and 50% comparing to $W_U^{(n)}$, $S_U^{(n)}$ and $S_U^{(n)}/W_U^{(n)}$, respectively.

More detailed analysis of considered effects might be done using Figs. 12, 13, and 14. They show significant increase of entropy with the increase of contact pressure, friction coefficient, and stresses caused by non-contact loads. Entropy increases almost at the same level for the same absolute values of tensile and compressive non-contact stresses. This effect might be conditioned by the fact that energy U calculated according to (25) attains positive values.

Main conclusion that can be made from the analysis of Figs. 8, 9, 10, 11, 12, 13, and 14 is that not only friction but also non-contact forces change greatly entropy characteristics in the neighborhood of contact area.

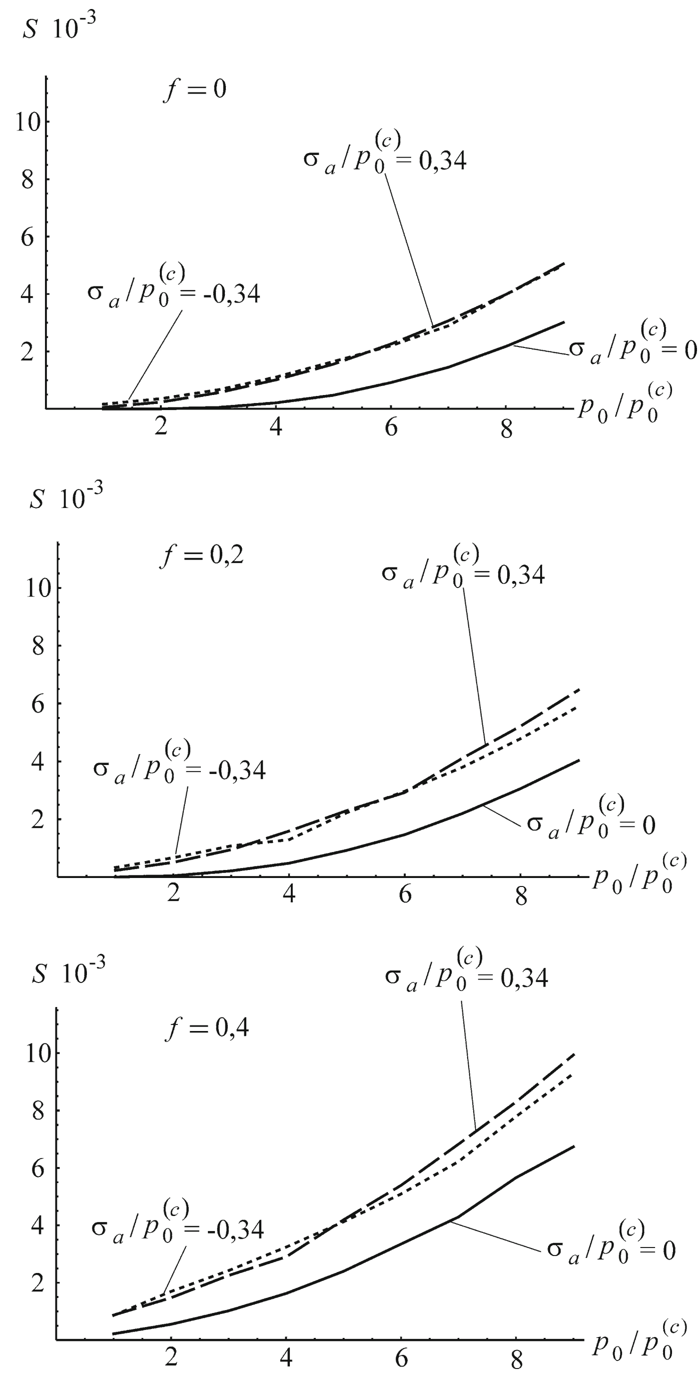

Fig. 14 Dependence of entropy on contact stresses

Note that according to (23a), (25)–(30) calculations were performed for the simplest case of fully applied to the tribo-fatigue system energy U. Similar calculations may be done for effective energies $U^{(\mathrm{eff})}$ taking into account damages interaction function Λ according to (19).

6 Conclusion

Let us make necessary generalizations.

If the evolution is analyzed from the combined viewpoint of the mechanothermodynamic state of the system, then with the regard to the aforesaid it can be understood: any system is threatened not with the thermodynamic death but with the damage and decomposition into components which in turn can and must be considered as initial elements to form and develop new systems, whose mode of existence is the motion and new damages. Hence, the evolution appears to be one-directed in time and, in the essence, infinite since the matter as the mode of its existence—motion—is indestructible.

Thus, as the first principle of mechanothermodynamics the statement should be taken: the degree of damageability of a system can be as large as desired:

$$\vec{\omega}_{\sum} = \vec{\omega}_{\sum}\left(U_{\sum}^{\text{eff}}\right) \overset{\vec{t}}{\Rightarrow} \infty \tag{31}$$

or the damageability of all living beings has no conceivable borders.

Then, it is possible to formulate the consequence of the first principle of mechanothermodynamics: production of internal mechanothermodynamic entropy is also eternal as well as motion and damage.

In the main, the first principle of mechanothermodynamics can be treated as the assertion: the entropy of the Universe increases. In his known lectures on physics [12], Feynman formulated the second principle of thermodynamics in the same manner. He proceeded from the following argument: the system of Universe type is always peculiar to irreversible thermodynamic changes.

Taking into consideration (19) and (20), it is possible to formulate the second principle of mechanothermodynamics: flows of effective energy (entropy) caused by different-nature sources under irreversible changes in the mechanothermodynamic system are not summed up—they interact in a complex manner.

Such Λ-interactions are described by the expressions [6]

$$\begin{aligned} U_{\sum}^{\text{eff}} &= U_{\sum}^{\text{eff}}\left(\Lambda_1, \ldots, \Lambda_m, U_1^{\text{eff}}, \ldots, U_n^{\text{eff}}\right), \quad m < n, \\ S_i &= S_i\left(\Lambda_1, \ldots, \Lambda_m, S_i^{(1)}, \ldots, S_i^{(n)}\right), \quad m < n. \end{aligned} \tag{32}$$

As shown above, the result of a diversity of Λ-interactions is the development (accumulation) of internal damages in the system elements which are determined by the unity and the struggle of opposite processes of physical hardening–softening. Therefore the interaction functions should belong to three classes of values $\Lambda \gtreqless 1$).

Thus, based on (5–10, 13–22) one can develop the fundamentals of mechanothermodynamics, just as using (1)–(4) thermodynamics (see, example, [1]) has been created.

Formulas (22)–(25) allow effective practical analysis of entropy for different sets of loads that is illustrated by Figs. 8, 9, 10, 11, 12, 13, 14.

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